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Theory of the Effects of Small Gravitational Levels on Droplet Gasification

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# THEORY OF THE EFFECTS OF SMALL GRAVITATIONAL LEVELS ON DROPLET GASIFICATION

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### ABSTRACT

A mathematical model taking into account small (and constant) gravitational levels is developed for vaporization of an isolated liquid droplet suspended in a stagnant atmosphere. A goal of the present analysis is to see how gravitational levels affect droplet gasification characteristics. Attention is focused upon determining effects on gas-phase phenomena. The conservation equations are normalized and nondimensionalized, and a small parameter that accounts for the effects of gravity is identified. This parameter is the square of the inverse of a Froude number based on the gravitational acceleration, the droplet radius, and a characteristic gas-phase velocity at the droplet surface. Asymptotic analyses are developed in terms of this parameter.

In the analyses, different spatial regions are identified. Near a droplet, gravitational effects are negligible in the first approximation, and the flowfield is spherically symmetric to the leading order. Analysis shows, however, that outer zones exist where gravitational effects cannot neglected; it is expected stagnation point will be present in an outer zone that is not present when gravity is totally absent. The leadingorder and higher-order differential equations for each zone are derived and solved. The solutions allow the effects of gravity on vaporization rates and temperature, velocity and species

fields to be determined.

### NOMENCLATURE

D mass diffusion coefficient

internal energy

$$(=\sum_{i=1}^{N}h_{i}Y_{i}-P/\rho)$$

normalized gravitational vector

Fr Froude number (=  $U_s / \sqrt{gR}$ )

gravitational acceleration

heat of vaporization

Le Lewis number (=  $\lambda_g/(\rho_g C_p D_g)$ )

Ma Newtonian Mach number

m evaporation rate

gas-phase pressure

instantaneous droplet radius

Re Reynolds number  $= U_s R / V_{g,\infty}$ 

R gas constant

Sc Schmidt number (=  $v_{g,\infty}/D_{g,\infty}$ )

gas-phase velocity gas-phase temperature

U<sub>a</sub> radial velocity at the surface of

the droplet (= 
$$\frac{\rho_d}{\rho_g} \frac{k}{8R}$$
)

Vi diffusion velocity of species i

Y<sub>i</sub> mass fraction of species i

z contracted radial coordinate

n contracted radial coordinate  $(= \epsilon^{\frac{1}{3}}r)$ 

τ stress tensor

- Richardson number (=  $gR/U_s^2$ )
- viscous dissipation of energy
- y ratio of specific heats  $(= C_p / C_v)$
- v kinematic viscosity
- u dynamic viscosity
- gas-phase density

# Subscripts and Superscripts

- dimensional quantity
- outer expansion variable (zone II)
- outer expansion variable (zone III)
- gas phase
- d
- liquid phase radial direction
- droplet surface
- leading order quantity
- higher order quantity
- ∞ far field state

### INTRODUCTION

The last few decades have seen an increasing interest in theory experiments on evaporation combustion of a liquid droplet. This interest reflects the importance of this area to the understanding of the fundamentals of combustion science. It also reflects the fact that a significant current practical portion of applications use liquid fuels as a source of energy due to convenience in transportation and storage. In most of these applications spray combustion is used, where liquid fuel would exist as droplets inside a combustion chamber.

A detailed study of spray combustion is difficult due to the complexity of interactions between droplets. One logical way to gain an understanding of the general behavior of spray to study droplet phenomena is vaporization and/or combustion under When well-controlled conditions. experiment and theory agree for simplified situations, predictions may be made with increased confidence for more complex cases where accurate experimental data may not exist. For this reason, many studies have focused upon isolated droplets undergoing spherically symmetrical vaporization with and without combustion.

Reducing the acceleration of gravity to negligible levels is one of the most critical conditions for isolated droplet experiments that strive for spherical symmetry. Based on this condition, the spherically symmetric vaporization and combustion assumption can be applied provided that there is no forced convection or any interaction between the droplet and its environment other than heat and mass transfer. However, gravitational reducing the acceleration is not a simple task and a nonzero effective gravity level is invariably present in these type of experiments. The effects of small gravitational levels on droplet phenomena are largely unknown. Therefore, studies to model the effects of gravity on the vaporization and combustion of isolated droplets are warranted. These studies can increase understanding of the importance of small buoyancy levels and in turn will help in the interpretation of results from microgravity experiments.

In this study, we will develop a mathematical model using asymptotic expansions for an isolated liquid droplet suspended in a stagnant oxidizing atmosphere which undergoes diffusion dominated vaporization, taking into account gravity effects. In droplet many recent theoretical models, buoyancy effects have been neglected. In contrast, gravity levels are never zero in experiments. For example, the effective gravity level can be reduced to  $10^{-6}$  m/s<sup>2</sup> (at best) in the space shuttle (though accelerations of about 10-3 m/s<sup>2</sup> are more typical), to 10-5 m/s<sup>2</sup> in drop-towers, and to 10-3 m/s<sup>2</sup> in a drop-tube apparatus<sup>1</sup> (Wang, D. and Shaw, B. D., 1991). Depending on the droplet diameter these gravity levels can be important and might significantly affect outcomes of the experiments. There are quite a few published articles that cover various aspect of droplet evaporation and combustion, but there is a no published source that looked at the buoyancy effects within this frame work. In this study, we hope to shed light on this factor and provide a basic understanding of its importance.

# THE GOVERNING EQUATIONS

The dimensional governing equations can be written as follows:

$$\partial \rho^* / \partial t^* + \vec{\nabla} \cdot (\rho^* \vec{u}^*) = 0$$
 (continuity)

$$\begin{split} & \partial Y_{i} / \partial t^{*} + \vec{u}^{*} \cdot \vec{\nabla} Y_{i} \\ & = \frac{1}{2} \left[ \vec{\nabla} \cdot \left( \rho^{*} Y_{i} V_{i}^{*} \right) \right] / \rho^{*} \end{split} \tag{species}$$

$$\rho^* \partial e^* / \partial t^* + \rho^* \vec{u}^*. \vec{\nabla} e^*$$

$$= \nabla . (\lambda^* \nabla T^*) - P^* (\nabla . u^*) \qquad (energy)$$

$$\rho^* \partial \vec{u}^* / \partial t^* + \rho^* \vec{u}^*. \vec{\nabla} \vec{u}^*$$

$$= - \vec{\nabla} P^* + \vec{\nabla} . \vec{\tau}^* + (\rho^* - \rho_{\infty}^*) g^* \pmod{\text{momentum}}$$

$$p^* = \rho^* \Re T^*$$
 (equation of state)

where:

$$\begin{aligned} h_i^* &= h_i^{*o} + \int\limits_{T^o}^T C_{p,i}^* \ dT^* \\ Let \, \nabla \Phi &= \rho_\infty^* \ g^* \\ P^* &= p^* - \Phi \end{aligned}$$

These equations will be nondimensionalized by using the following variables:

$$\begin{split} u &= \frac{u^{\star}}{U_{s}^{\star}} \quad , \quad \rho &= \frac{\rho^{\star}}{\rho_{\infty}^{\star}} \quad , \quad T &= \frac{T^{\star}}{T_{\infty}^{\star}} \\ r &= \frac{r^{\star}}{R^{\star}} \quad , \quad P &= \frac{P^{\star}}{P_{\infty}^{\star}} \quad , \quad L &= \frac{L^{\star}}{C_{p}^{\star} T_{\infty}^{\star}} \\ D &= \frac{D^{\star}}{D_{\infty}^{\star}} \quad , \quad g &= \epsilon = \frac{g^{\star}}{(U_{s}^{\star 2}/R^{\star})} = \left(\frac{1}{Fr^{2}}\right) \end{split}$$

where U<sub>8</sub>\* is the radial velocity at the surface of a droplet undergoing purely spherically symmetrical combustion in the absence of gravity.

### **ASSUMPTIONS**

For a liquid fuel droplet vaporizing in an oxidizing atmosphere of an insoluble gaseous species, the above equations are simplified according to the following assumptions:

Quasi-steady state vaporization, so the time-dependent terms can be neglected. Taking the molecular weights and the specific heats to be constants, and the product pD to be constant. Constant gasphase transport properties. Lewis and Schmidt numbers are constants. Perfect gas. Single fuel species droplet. No spray effects, that is the liquid fuel droplet is isolated and suspended in a stagnant environment. Constant thermodynamic pressure. Constant and uniform droplet temperature. The Newtonian number is small, M<sup>2</sup><< 1. Fick's law for mass diffusion (  $V_i = -D\nabla \ln Y_i$ ) holds.

Using these assumptions as well as the nondimensional variables allows the governing equations to be nondimensionalized as follows:

$$\vec{\nabla} \cdot (\rho \vec{\mathbf{u}}) = 0$$
 (continuity)

Re Sc 
$$\rho \bar{\mathbf{u}} \cdot \bar{\nabla} Y_i$$

$$= \bar{\nabla}^2 Y_i \qquad \text{(species)}$$

Re Sc 
$$\rho \vec{u}$$
 .  $\vec{\nabla} T$  - Le  $\vec{\nabla}^2 T$   
=  $(\gamma - 1)M^2(\phi + \vec{u}.\vec{\nabla} p)$  (energy)

$$\rho \vec{u} \cdot \vec{\nabla} \vec{u} + \frac{\vec{\nabla} p}{Ma^2}$$

$$= \frac{\vec{\nabla} \cdot \vec{\tau}}{Re} + \epsilon (\rho - 1) \vec{f} \qquad \text{(momentum)}$$

$$P = \rho T$$
 (state)

The dimensionless boundary conditions are:

at 
$$r = 1$$

$$u_{\theta} = 0$$

$$T = T_s$$

$$\sum_{i=1}^{N} Y_i = 1$$

$$Y = \exp \left[ -\frac{\gamma L}{\gamma - 1} \left( \frac{T_b - T_s}{T_b T_s} \right) \right]$$

$$\frac{\partial Y_i}{\partial r}\Big|_s = \frac{\text{Re Sc } u_r}{D} (Y_{is} - 1)$$

$$\frac{\partial T}{\partial r}\Big|_s = \frac{\text{Re Sc } u_r L}{\text{Le D}}$$

and as  $r \rightarrow \infty$ 

$$u_r \rightarrow 0$$

$$u_{\theta} \rightarrow 0$$

$$Y_i \rightarrow Y_{im}$$

$$T \rightarrow 1$$

$$\rho \rightarrow 1$$

$$\frac{\nabla P}{Ma^2} = \epsilon(\rho - 1)\vec{f}$$

### ANALYSIS TECHNIQUE

The technique of matched asymptotic expansions will be used as a tool to study the effects of gravity on the flow variables during the evaporation of a liquid fuel droplet.

There have been a number of studies of droplet evaporation and combustion using this perturbation technique. For example, Fendell, Spankle and Dodson<sup>2</sup> (1966) used a singular perturbation consider droplet approach to combustion with convective flow field that is entirely in the Stokes regime. A Reynolds number was used as their small parameter of expansion. Fendell, Coats and Smith also studied droplet vaporization in a slow, compressible and viscous flow<sup>3</sup>. Kassoy et al.<sup>4</sup> (1966), studied compressible low Reynolds number flow around a sphere. They derived inner and outer expansions for the flow variables in terms of the difference, Reynolds temperature number and the free stream Mach number. Acrivos and Taylor<sup>5</sup> derived an expansion in terms of a small temperature difference between a sphere and free stream and calculated the average Nusselt number of the incompressible flow over a with a Stokes velocity profile. Gogos et al.6 studied evaporation of a fuel droplet with a strong evaporationradial velocity induced undergoing slow translation. introduced the translation otherwise perturbation to an stationary droplet. Gogos et al. have also considered combustion of droplets undergoing slow translation?. Mahoney<sup>8</sup>, Fendell<sup>9</sup>, and Sato<sup>10</sup> have presented asymptotic analyses of natural convection heat transfer from rigid spheres assuming that the Grashof number is small, while Hieber Gebhart 1 1 studied convection about rigid spheres in the limit of small gravity effects.

In the model presented here, a quasisteady state assumption is used because of the fact that the ratio between the densities of the gas-phase and of the droplet is small  $O(10^{-3})$ . Matched asymptotic expansions are used based on a small parameter identified as the ratio of the inertia force to the gravity force. This parameter is the inverse of the Froude number  $(U_s^*/\sqrt{g^*R^*})$ 

squared, where  $U_s$  is the droplet surface velocity, R is the instantaneous droplet radius, and  $g_s$  is the gravity acceleration. This parameter is the variable  $\varepsilon$  that appears in the dimensionless momentum equation.

### PHYSICAL CONSIDERATIONS

An order-of-magnitude analysis of the governing equations indicated the relative importance of each term in the equations. It was found that the order of magnitude of the nondimensional gravity term in the momentum equation was a useful guide to distinguish the different governing physical characteristics. This term was equal to the reciprocal of the Froude number squared and is denoted as  $\varepsilon = g^* R / U_s^{*2}$ , where  $\varepsilon << 1$ . For typical situations, a 1 mm droplet has values of E ranges from 10<sup>-4</sup> in drop-tube apparatus to 10<sup>-6</sup> in space shuttle experiments. In order to study the flow field around a droplet, the governing equations have to be solved. These equations in their original form are very complex. Simplifying assumptions have to be made in order for one to approach these equations analytically. instance: the thermodynamic pressure was considered to be constant since characteristic Mach numbers (M) for the flow are significantly less than unity. The pressure gradient in the momentum equation, on the other hand, can not be neglected since it is divided by (M<sup>2</sup>) which is very small.

The non-dimensional equation of state is then simply  $\rho T = 1$ . In the energy equation the second term of the right hand side can be neglected. This dissipation term is of  $O(M^2)$ , where M << 1. Taking the Lewis number as constant will allow us to have  $\rho D$ =constant.

A physical approach was used to deduce that this problem is singular for  $\varepsilon \to 0$ , as well as the expansions for inner and outer regions. In an inner region near the droplet (termed zone (I)), the first in the expansion is spherically symmetric solution. solution will prevail in the absence of any significant gravity effects and the flowfield will only vary in the radial direction. For the next term in the expansion, a higher-order correction of  $O(\epsilon)$  will be applicable to the velocity field, which is a result of the body force term. From the continuity equation it can be seen that the order of magnitude change in density is as large as the change in velocity. From the equation of state, the order of magnitude changes in temperature will be as large as the changes in density. Therefore, it was concluded that the  $O(\varepsilon)$  correction is applicable to all of the flow variables (velocity, density, mass fraction, temperature and pressure). To locate zone (II) i.e., where gravity is important, we had to determine where the sphericallysymmetrical solution breaks down. From the leading-order continuity equation, the velocity field was obtained. This velocity field was then used in the radial momentum equation to estimate where the largest term of the equation becomes of the same order as the gravity term, i.e.  $O(\varepsilon/r)$ . It was found that the largest term of the inertial, pressure and viscous terms is of  $O(1/r^5)$ . Thus, it was concluded that the negligible buoyancy assumption is no longer valid when  $(1/r^4) \approx \varepsilon$ , and that gravity should be accounted for when  $r = 1/\epsilon^{1/4}$ . The rescaled radial coordinate  $z = \epsilon^{1/4}r$  was introduced and expansions for all flow variables were obtained.

### SOLUTION METHOD

The method of matched asymptotic expansions is used in this study to solve the governing equations that describe the flow field around the droplet. Inner and outer expansions are derived for each of the flow variables, density, mass (velocity, temperature and pressure), in terms of a small parameter (E). The solutions are typically developed up to the second terms of the expansions. The basic simplification of the governing equations comes from the assumption that E<<1 where as stated above, the spherically symmetric assumption is applicable when  $(1/r^4) >> \epsilon$ , since when  $(1/r^4) \approx \varepsilon$  the effect of buoyancy would be felt in the flow field and this will invalidate this assumption.

In zone (I), the gravity term suggests an  $O(\varepsilon)$  correction to the velocity field. This correction is also applicable to the other dependent variables.

The inner expansions valid near the droplet are as follows:

$$u(r,\theta;\epsilon) = u_0 + \epsilon u_1 + \dots$$

$$\rho(r,\theta;\epsilon) = \rho_0 + \epsilon \rho_1 + \dots$$

$$Y(r,\theta;\epsilon) = Y_0 + \epsilon Y_1 + \dots$$

$$T(r,\theta;\epsilon) = T_0 + \epsilon T_1 + \dots$$

$$P(r,\theta;\epsilon) = P_0 + \epsilon P_1 + \dots$$
where:
$$u_1 = [u_{r10}(r) + u_{r11}(r)\cos\theta]\hat{e}_r + [u_{\theta 11}(r)\sin\theta]\hat{e}_\theta$$

$$\rho_1 = \rho_{10}(r) + \rho_{11}(r)\cos\theta$$

$$Y_1 = Y_{10}(r) + Y_{11}(r)\cos\theta$$

$$T_1 = T_{10}(r) + T_{11}(r)\cos\theta$$

$$P_1 = P_{10}(r) + P_{11}\cos\theta$$

Substituting the above expansions into the governing equations produces the following equations:

Order  $\varepsilon^0$ :

$$\vec{\nabla} \cdot (\rho_0 \vec{u}_0) = 0$$

$$\text{Re Sc } \rho_0 \vec{u}_0 \cdot \vec{\nabla} Y_0 = \vec{\nabla}^2 Y_0$$

$$\frac{\text{Re Sc}}{\text{Le}} \rho_0 \vec{u}_0 \cdot \vec{\nabla} T_0 = \vec{\nabla}^2 T_0$$

$$\rho_0 \vec{u}_0 \cdot \vec{\nabla} \vec{u}_0 = -\frac{\vec{\nabla} p_0}{\text{Ma}^2} + \frac{\vec{\nabla} \cdot \vec{\tau}_0}{\text{Re}}$$

$$\rho_0 T_0 = 1$$

Boundary conditions:

at r = 1  

$$u_{r0} = 1$$
  
 $u_{\theta 0} = 0$   
 $T_0 = T_s$   
 $Y = \exp \left[ -\frac{\gamma_L}{\gamma - 1} \left( \frac{T_b - T_s}{T_b T_s} \right) \right]$   
 $\frac{\partial Y_o}{\partial r} \bigg|_s = \frac{\text{Re Sc } u_{ro}}{D} (Y_s - 1)$   
 $\frac{\partial T_o}{\partial r} \bigg|_s = \frac{\text{Re Sc } L}{\text{Le D}} u_{ro}$ 

The analytical solution to the above spherically symmetric ordinary differential equations is:

$$\begin{split} u_{r0} &= \frac{m_0}{\rho_0 \, r^2} \\ \rho_0 &= \frac{1}{T_0} \\ T_0 &= T_s - L + L \bigg[ exp \bigg( \frac{ReSc \, m_0}{Le} (1 - \frac{1}{r}) \bigg) \bigg] \\ Y_0 &= 1 + (Y_\infty - 1) \, exp \bigg[ - ReSc \frac{m_0}{r} \bigg] \\ \frac{dP_0}{dr} &= Ma^2 \bigg\{ 2 \frac{m_0^2}{r^5} T_0 - \frac{2m_0^2}{r^3} \frac{dT_0}{dr} \\ &+ \frac{8}{3Re} \bigg[ \frac{m_0}{r^2} \frac{d^2 T_0}{dr^2} - \frac{m_0}{r^3} \frac{dT_0}{dr} \bigg] \bigg\} \end{split}$$

where mo is the integration constant

Order  $\varepsilon^1$ :

$$\vec{\nabla} \cdot (\rho_0 \vec{\mathbf{u}}_1 + \rho_1 \vec{\mathbf{u}}_0) = 0$$

Re Sc 
$$((\rho_0 \vec{\mathbf{u}}_1 + \rho_1 \vec{\mathbf{u}}_0) \cdot \vec{\nabla} \mathbf{Y}_0 + \rho_0 \vec{\mathbf{u}}_0 \cdot \vec{\nabla} \mathbf{Y}_1) = \vec{\nabla}^2 \mathbf{Y}_1$$

$$\begin{split} &\frac{\text{Re Sc}}{\text{Le}} \left( (\rho_0 \vec{\mathbf{u}}_1 \, + \, \rho_1 \vec{\mathbf{u}}_0) \, . \, \, \vec{\nabla} T_0 \right. \\ &+ \left. \rho_0 \vec{\mathbf{u}}_0 \, . \, \, \vec{\nabla} T_1 \right) = \vec{\nabla}^2 T_1 \end{split}$$

$$\rho_0 \vec{u}_0 \cdot \vec{\nabla} \vec{u}_1 + (\rho_0 \vec{u}_1 + \rho_1 \vec{u}_0) \cdot \vec{\nabla} \vec{u}_0$$

$$= -\frac{\vec{\nabla} P_1}{Ma^2} + \frac{\vec{\nabla} \cdot \vec{\tau}_1}{Re} + (\rho_0 - 1)\vec{f}$$

$$\rho_1 = -T_1$$

The above equations will actually consist of two sets of equations:

$$\begin{split} &\frac{d}{dr} \left[ r^2 \left( \rho_0 u_{r10} + \rho_{10} u_{r0} \right) \right] = 0 \quad \text{(continuity)} \\ &\frac{m_0}{r^2} \frac{du_{r10}}{dr} + \frac{m_{10}}{r^2} \frac{du_{r0}}{dr} + \frac{1}{Ma^2} \frac{dP_{10}}{dr} \\ &= \frac{1}{Re} \left[ \frac{4}{3} \frac{d^2 u_{r10}}{dr^2} + \frac{8}{3} \frac{d}{dr} \left( \frac{u_{r10}}{r} \right) \right] \text{(momentum)} \end{split}$$

$$\begin{split} &\frac{\text{ReSc}}{\text{Le}} \bigg( \frac{m_0}{r^2} \frac{dT_{10}}{dr} + \frac{m_{10}}{r^2} \frac{dT_0}{dr} \bigg) \\ &- \frac{1}{r^2} \frac{d}{dr} \bigg( r^2 \frac{dT_{10}}{dr} \bigg) = 0 \end{split} \tag{energy}$$

$$\begin{split} &\text{ReSc}\bigg(\frac{m_0}{r^2}\frac{dY_{10}}{dr} + \frac{m_{10}}{r^2}\frac{dY_0}{dr}\bigg) \\ &-\frac{1}{r^2}\frac{d}{dr}\bigg(r^2\frac{dY_{10}}{dr}\bigg) = 0 \end{split} \qquad \text{(species)} \end{split}$$

$$\rho_{10} = -\rho_0^2 T_{10} \qquad \qquad \text{(state)}$$

$$at r = 1$$
  
 $Y_{i10} = 0$   $T_{10} = 0$ 

$$\frac{\partial Y_{i10}}{\partial r}\Big|_{s} = -\frac{\text{Re Sc}}{D} [u_{r10}(1-Y_{0}) + u_{r0}Y_{10}]$$

$$\frac{\partial T_1}{\partial r}\bigg|_{c} = \frac{\text{Re Sc L}}{\text{Le D}} u_{r10}$$

Applying the method of the integrating factor to the energy and species equations, we get:

$$\begin{split} T_{10}(r) &= -\frac{m_{10}}{m_0} (T_s - L) + \frac{Le \, C_1}{Re \, Scm_0} \\ &- \frac{Re \, Sc}{Le} \, L \bigg( \frac{m_{10}}{r} \bigg) exp \bigg[ \frac{Re \, Sc}{Le} \bigg( m_0 - \frac{m_0}{r} \bigg) \bigg] \\ &+ C_{11} \, exp \bigg( -\frac{Re \, Sc}{Le} \frac{m_0}{r} \bigg) \end{split}$$

where  $C_1$  and  $C_{11}$  are constants

$$Y_{10}(r) = -\frac{m_{10}}{m_0} + \frac{C_2}{ReScm_0}$$

$$-ReSc\left(\frac{m_{10}}{r}\right)(Y_{\infty} - 1)exp\left(-ReSc\frac{m_0}{r}\right)$$

$$+C_{22}exp\left(-ReSc\frac{m_0}{r}\right)$$
where  $C_2$  and  $C_{22}$  are constants

Evaporation Rate Modification:

Let 
$$\mu = \cos\theta$$

Then the evaporation rate can be defined by:  $\dot{m} = 2\pi r^2 \int \rho u_r d\mu$ ,

substituting the inner expansions into the above equation, we get:

$$\dot{m} = 2\pi r^2 \left[ 2\rho_0 u_{r0} + 2\epsilon \left( \rho_0 u_{r10} + \rho_{10} u_{r0} \right) + \ldots \right]$$

comparing the above equation to the spherically symmetric rate of evaporation,  $\dot{m}=4\pi(r^2\rho_0u_{r0})$ , we get :

$$\frac{\dot{m}}{\dot{m}_0} = 1 + \varepsilon \left[ \frac{\left( \rho_0(r) u_{r10}(r) + \rho_{10}(r) u_{r0}(r) \right)}{\rho_0(r) u_{r0}(r)} \right]$$

at r = 1, 
$$u_{r0} = 1$$
  

$$\Rightarrow \frac{\dot{m}}{\dot{m}_0} = 1 + \varepsilon \left[ \frac{\left( \rho_0(1) u_{r10}(1) + \rho_{10}(1) \right)}{\rho_0(1)} \right]$$

From continuity in the first order problem we have:

$$\begin{split} &\frac{m_{10}}{r^2} = \rho_0(r) u_{r10}(r) + \rho_{10}(r) u_{r0}(r) \\ &\Rightarrow m_{10} = \left(\rho_0(1) u_{r10}(1) + \rho_{10}(1)\right) \end{split}$$

Where  $m_{10}$  can be found from the solution of the energy or the species equations for r = 1.

By substituting the second part of the higher order expansion we get coupled boundary value problems that can be solved numerically.

$$\frac{du_{r11}}{dr} = -\left(\frac{2}{r} + \frac{1}{\rho_0} \frac{d\rho_0}{dr}\right) u_{r11} + \frac{m_0}{r^2} \frac{dT_{11}}{dr} + \frac{m_0}{\rho_0} \frac{T_{11}}{r^2} \frac{d\rho_0}{dr} - 2 \frac{u_{\theta 11}}{r} = 0 \text{ (continuity)}$$

$$\frac{d^2 u_{\theta 11}}{dr^2} = \left(\frac{m_0}{r^2} - \frac{2}{r}\right) \frac{du_{\theta 11}}{dr} - \frac{1}{r} \frac{du_{r11}}{dr} + \left[-\frac{P_{11}}{r^2} + \frac{4}{3} \left(\frac{du_{r11}}{dr} - \frac{u_{r11} + u_{\theta 11}}{r}\right)\right]$$

$$+\left(\frac{m_0}{r^3} - \frac{4}{r^2}\right)u_{\theta 11}$$

$$-4\frac{u_{r11}}{r^2} + \rho_0 \qquad (\theta - \text{momentum})$$

$$a_{11} = -\frac{P_{11}}{Ma^2} + \frac{4}{3} \left( \frac{du_{r11}}{dr} - \frac{u_{r11} + u_{\theta 11}}{r} \right)$$

$$\begin{split} \frac{da_{11}}{dr} &= \left(\frac{m_0}{r^2} - \frac{4}{r}\right) \frac{du_{r11}}{dr} - \frac{2}{r} \frac{du_{\theta 11}}{dr} \\ &+ \left(\rho_0 \frac{du_{r0}}{dr} + \frac{6}{r^2}\right) u_{r11} + \frac{6}{r^2} u_{\theta 11} \\ &- \left(T_{11} \frac{m_0}{r^2}\right) \frac{du_{r0}}{dr} - \rho_0 \quad \text{(r-momentum)} \end{split}$$

$$\begin{split} &\frac{d^2T_{11}}{dr^2} = \left(\frac{2}{r^2} - \frac{Re\,Sc}{Le} \frac{m_0}{r^2} \rho_0\right) T_{11} \\ &+ \left(\frac{Re\,Sc}{Le} \frac{m_0}{r^2} - \frac{2}{r}\right) \frac{dT_{11}}{dr} \\ &+ \frac{Re\,Sc}{Le} u_{r11} \rho_0 \frac{dT_0}{dr} \quad \text{(energy)} \\ &\frac{d^2Y_{11}}{dr^2} = \left(\frac{2}{r^2} - Re\,Sc \frac{m_0}{r^2} \rho_0 \frac{dT_0}{dr}\right) Y_{11} \\ &+ \left(Re\,Sc \frac{m_0}{r^2} - \frac{2}{r}\right) \frac{dY_{11}}{dr} \\ &+ Re\,Sc u_{r11} \rho_0 \frac{dY_0}{dr} \quad \text{(species)} \end{split}$$

Numerical solutions of these equations will be reported at a later time.

In zone (II), the expansions for the flow variables will be of the following form:

$$\begin{split} \tilde{\mathbf{u}}(z,\theta;\epsilon) &= \sum_{\mathbf{n}=0} \beta_{\mathbf{n}} \tilde{\mathbf{u}}_{\mathbf{n}}(z,\theta;\epsilon) \\ \tilde{\rho}(z,\theta;\epsilon) &= \sum_{\mathbf{n}=0} \alpha_{\mathbf{n}} \tilde{\rho}_{\mathbf{n}}(z,\theta;\epsilon) \\ \tilde{\mathbf{Y}}(z,\theta;\epsilon) &= \sum_{\mathbf{n}=0} \delta_{\mathbf{n}} \tilde{\mathbf{Y}}_{\mathbf{n}}(z,\theta;\epsilon) \\ \tilde{\mathbf{T}}(z,\theta;\epsilon) &= \sum_{\mathbf{n}=0} \chi_{\mathbf{n}} \tilde{\mathbf{T}}_{\mathbf{n}}(z,\theta;\epsilon) \\ \tilde{\mathbf{P}}(z,\theta;\epsilon) &= \sum_{\mathbf{n}=0} \eta_{\mathbf{n}} \tilde{\mathbf{P}}_{\mathbf{n}}(z,\theta;\epsilon) \end{split}$$

where the coefficients  $\beta_n$ ,  $\alpha_n$ ,  $\delta_n$ ,

 $\chi_n$  , and  $\eta_n$  are, in general, functions of E. To determine the first coefficient of the above expansion, the zeroth. order solution of the inner expansion substituted into the component of the momentum equation. It was found that the viscous, pressure convective-acceleration were O(1/r<sup>5</sup>), while buoyancy effects were  $O(\varepsilon/r)$ . Therefore, the buoyancy effects are negligible as long as  $(1/r^4) \gg \epsilon$ ; when  $(1/r^4) \approx \epsilon$  the above assumption is no longer valid. An outer variable (z) is introduced as a contracted coordinate, where  $z = \varepsilon^{4}$ r. the leading-order momentum equation, the density appears as a constant. In a constant-density flow field, the buoyancy will have no effects on the velocity field and therefore, the leading-order flowfield is spherically symmetrical.

To determine the appropriate higher order corrections for the dependent variables in zone (II), the leadingcomposite solution substituted into the governing equations. The terms were expanded. The momentum equation was found to be satisfied to leading order, with an error of  $O(\epsilon^{\frac{1}{4}})$ . This suggested that the higher order correction terms for the flow variables in zone (II) are smaller than the leading-order terms. Thus, the coefficients then are:

$$\beta_0 = \varepsilon^{2/4}$$
,  $\alpha_0 = \delta_0 = \chi_0 = 1$ ,  $\eta_0 = \varepsilon$   
 $\beta_1 = \varepsilon^{3/4}$ ,  $\alpha_1 = \delta_1 = \chi_1 = \varepsilon^{1/4}$ ,  $\eta_1 = \varepsilon$ 

And the outer expansions which are valid far away from the droplet are:

$$z = \varepsilon^{1/4} r$$

$$\vec{\nabla} = \varepsilon^{1/4} \vec{\nabla}$$

$$\vec{u}(z,\theta; \varepsilon) = \varepsilon^{2/4} \vec{u}_0 + \varepsilon^{3/4} \vec{u}_1 + \dots$$

$$\vec{\rho}(z,\theta; \varepsilon) = 1 + \varepsilon^{1/4} \vec{\rho}_0 + \varepsilon^{2/4} \vec{\rho}_1 + \dots$$

$$\vec{Y}(z,\theta; \varepsilon) = Y_{\infty} + \varepsilon^{1/4} \vec{Y}_0 + \varepsilon^{2/4} \vec{Y}_1 + \dots$$

$$\vec{T}(z,\theta; \varepsilon) = 1 + \varepsilon^{1/4} \vec{T}_0 + \varepsilon^{2/4} \vec{T}_1 + \dots$$

$$\vec{P}(z,\theta; \varepsilon) = \varepsilon \vec{P}_0 + \varepsilon^{5/4} \vec{P}_1 + \dots$$

Equations for the correction terms were derived by substituting the outer expansions into the governing equations and then grouping terms of equal order of magnitude.

# Zone (II) leading-order equations:

$$\begin{split} &\vec{\tilde{\nabla}} \cdot \vec{\tilde{u}}_0 = 0 \\ &\vec{\tilde{\nabla}}^2 \tilde{T}_0 = 0 \\ &\vec{\tilde{\nabla}}^2 \tilde{Y}_0 = 0 \\ &\vec{\tilde{\nabla}} \cdot \tilde{\tilde{\tau}}_0 = 0 \\ &\tilde{\tilde{\rho}}_0 = -\tilde{T}_0 \end{split}$$

as 
$$r \to \infty$$
 $\tilde{u}_{z0} \to 0$ 
 $\tilde{u}_{\theta0} \to 0$ 
 $\tilde{T}_0 \to 0$ 
 $\tilde{Y}_0 \to 0$ 

In the leading-order equations, the solution is spherically symmetric and the effects of the buoyancy are not yet apparent. Only the radial velocity component is nonzero; this component is shown below:

$$\tilde{\mathbf{u}}_{z0} = \frac{\tilde{\mathbf{m}}_0}{z^2}$$

Solutions to the energy and species equations are :

$$\tilde{T}_0 = \sum_{n=0}^{\infty} \frac{N_n}{z^{n+1}} P_n(\cos\theta)$$

$$\tilde{Y}_0 = \sum_{n=0}^{\infty} \frac{V_n}{z^{n+1}} P_n(\cos\theta)$$

where  $P_n$  is Legendre polynomial of order n.

Matching the leading order solutions: Inner leading - order solution as  $r \to \infty$ 

$$\begin{split} &u_{r} \sim \frac{m_{0}}{r^{2}} + O(r^{-3}) \\ &T \sim 1 - \frac{\text{Re Sc}}{\text{Le}} \frac{m_{0}}{r} \Big[ 1 - \big( T_{s} - L \big) \Big] + O(r^{-2}) \\ &Y \sim Y_{\infty} - \big( Y_{\infty} - 1 \big) \frac{\text{Re Sc } m_{0}}{r} + O(r^{-2}) \\ &\rho \sim 1 + \frac{\text{Re Sc}}{\text{Le}} \frac{m_{0}}{r} \Big[ 1 - \big( T_{s} - L \big) \Big] + O(r^{-2}) \\ &P \sim \frac{m_{0}^{2}}{2r^{4}} \Big\{ 1 - \frac{\text{Re Sc } m_{0}}{\text{Le}} \Big[ \big( T_{s} - L \big) - 1 \Big] \Big\} \\ &+ O(r^{-5}) \end{split}$$

Zone II leading - order solutions as  $z \rightarrow 0$ 

$$\tilde{u}_{z} \sim \varepsilon^{\frac{2}{5}} \left( \frac{\tilde{m}_{0}}{z^{2}} \right)$$

$$\tilde{T} \sim 1 + \varepsilon^{\frac{1}{5}} \left( \frac{N_{0}}{z} \right) + \dots$$

$$\tilde{Y} \sim Y_{\infty} + \varepsilon^{\frac{1}{5}} \left( \frac{V_{0}}{z} \right) + \dots$$

$$\tilde{\rho} \sim 1 - \varepsilon^{\frac{1}{5}} \left( \frac{N_{0}}{z} \right) + \dots$$

Matching the two solutions gives:

$$N_n = S_n = 0$$
 for  $n \ge 1$ 

$$\tilde{m}_0 = m_0$$

$$N_0 = -\frac{\text{Re Sc m}_0}{\text{Le}} [1 - (T_s - L)]$$

$$V_0 = -\text{ReSc } m_0(Y_{\infty} - 1)$$

The higher-order equations:

$$\vec{\tilde{\nabla}} \cdot (\tilde{\rho}_0 \vec{\tilde{u}}_0 + \vec{\tilde{u}}_1) = 0$$

$$\frac{\text{ReSc}}{\text{Le}} \tilde{\mathbf{u}}_0. \bar{\tilde{\nabla}} \tilde{\mathbf{T}}_0 = \bar{\tilde{\nabla}}^2 \tilde{\mathbf{T}}_1$$

$$\operatorname{ReSc}\tilde{\mathbf{u}}_0.\vec{\tilde{\nabla}}\tilde{\mathbf{Y}}_0=\vec{\tilde{\nabla}}^2\tilde{\mathbf{Y}}_1$$

$$\tilde{\mathbf{u}}_0.\tilde{\nabla}\tilde{\mathbf{u}}_0 + \frac{\tilde{\nabla}\tilde{\mathbf{P}}_0}{\mathbf{Ma}^2} = \frac{\tilde{\nabla}.\tilde{\boldsymbol{\tau}}_1}{\mathbf{Re}} + \tilde{\boldsymbol{p}}_0\tilde{\mathbf{f}}$$

$$\tilde{\rho}_1 = -\tilde{T}_1$$

as 
$$r \rightarrow \infty$$

$$\tilde{u}_{r1} \rightarrow 0$$

$$\tilde{u}_{e1} \rightarrow 0$$

$$\tilde{T}_1 \rightarrow 0$$

$$\tilde{Y}_1 \rightarrow 0$$

$$\frac{\tilde{\nabla}\tilde{P}_0}{Ma^2} = \tilde{\rho}_0 \vec{f}$$

Solutions to the energy and species equations are:

$$\tilde{T}_1 = \sum_{n=0}^{\infty} \frac{E_n}{z^{n+1}} P_n(\cos\theta) - \frac{\text{ReSc } \tilde{m}_0}{\text{Le}} \frac{N_0}{2z^2}$$

$$\tilde{Y}_1 = \sum_{n=0}^{\infty} \frac{Q_n}{z^{n+1}} P_n(\cos\theta) - \text{Re Sc } \tilde{m}_0 \frac{V_0}{2z^2}$$

where P<sub>n</sub> is Legendre polynomial of order n.

The continuity and the momentum equations can be solved using a form of a solution expressed in terms of the stream functions:

$$\tilde{\mathbf{u}}_{z1} = \frac{1}{z^2 \sin \theta} \frac{d\psi}{d\theta} + \frac{\mathbf{m}_0 \mathbf{N}_0}{z^3}$$

$$\tilde{\mathbf{u}}_{\theta 1} = -\frac{1}{z \sin \theta} \frac{d\psi}{dz}$$

 $\tilde{u}_{\theta 1} = -\frac{1}{z \sin \theta} \frac{d\psi}{dz}$ Substituting the above equations into the governing equations and taking the curl of the momentum equation yields the following:

$$D^2 \psi = \sin^2 \theta \left( \frac{N_0}{z} \right)$$

$$D = \frac{\partial^2}{\partial z^2} + \frac{\sin \theta}{z^2} \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right)$$

The general solution is found to be:

$$\Psi = \left(\frac{A_1}{z} + A_2 z + A_3 z^2 + A_4 z^4\right)$$

$$+\frac{N_0}{8}z^3$$
  $\sin^2\theta$ 

$$\hat{\mathbf{u}}_{z1} = 2\left(\frac{\mathbf{A}_1}{z^3} + \frac{\mathbf{A}_2}{z} + \mathbf{A}_3 + \mathbf{A}_4 z^2\right)$$

$$+\frac{N_0}{4}z\Bigg)\!\cos\theta+\frac{m_0N_0}{z^3}$$

$$\hat{\mathbf{u}}_{01} = \left(\frac{\mathbf{A}_1}{z^3} - \frac{\mathbf{A}_2}{z} - 2\mathbf{A}_3 - 4\mathbf{A}_4 z^2\right)$$

$$-\frac{3N_0}{8}z$$
 sin  $\theta$ 

where  $A_1, A_2, A_3$  and  $A_4$  are constants

From the principle minimum singularity  $^{12}$ , it can be concluded that  $A_4$  should be equal to zero. The velocity profiles will then be as follows:

$$\hat{\mathbf{u}}_{z1} = 2 \left( \frac{A_1}{z^3} + \frac{A_2}{z} + \frac{N_0}{4} z \right) \cos \theta + \frac{m_0 N_0}{z^3}$$

$$\hat{\mathbf{u}}_{\theta 1} = \left( \frac{A_1}{z^3} - \frac{A_2}{z} - \frac{3N_0}{8} z \right) \sin \theta$$

For the above solution, the first-order velocity is unbounded as z increases, while the leading-order velocity field decreases as  $1/z^2$ . Eventually, as z increases, the velocity correction term will become as large as the leading-order velocity term. When this happens, the solution in zone (II) breaks down, and more analysis is necessary. Another zone is necessary so that the solution can satisfy the outer boundary conditions.

### Zone (III):

The new scaling factor in this zone is predicted from the failure of the solution of zone (II). the expansion equations for the this region are:

$$\eta = \varepsilon^{1/3} \mathbf{r}$$

$$\vec{\nabla} = \varepsilon^{1/3} \hat{\vec{\nabla}}$$

$$\vec{\hat{\mathbf{u}}}(\eta, \theta; \varepsilon) = \varepsilon^{2/3} \hat{\vec{\mathbf{u}}}_0 + \varepsilon \hat{\vec{\mathbf{u}}}_1 + \dots$$

$$\hat{\rho}(\eta, \theta; \varepsilon) = 1 + \varepsilon^{1/3} \hat{\rho}_0 + \varepsilon^{2/3} \hat{\rho}_1 + \dots$$

$$\hat{\mathbf{Y}}(\eta, \theta; \varepsilon) = \mathbf{Y}_{\infty} + \varepsilon^{1/3} \hat{\mathbf{Y}}_0 + \varepsilon^{2/3} \hat{\mathbf{Y}}_1 + \dots$$

$$\hat{\mathbf{T}}(\eta, \theta; \varepsilon) = 1 + \varepsilon^{1/3} \hat{\mathbf{T}}_0 + \varepsilon^{2/3} \hat{\mathbf{T}}_1 + \dots$$

$$\hat{\mathbf{P}}(\eta, \theta; \varepsilon) = \varepsilon \hat{\mathbf{P}}_0 + \varepsilon^{4/3} \hat{\mathbf{P}}_1 + \dots$$

The leading-order equations are as follows:

$$\vec{\hat{\nabla}} \cdot (\vec{\hat{\mathbf{u}}}_0) = 0$$
$$\vec{\hat{\nabla}}^2 \hat{\mathbf{T}}_0 = 0$$

$$\vec{\hat{\nabla}}^2 \hat{Y}_0 = 0$$

$$\vec{\hat{\nabla}} \hat{P}_0 = \vec{\hat{\nabla}} \cdot \hat{\tau}_0 + \hat{\rho}_1 \vec{f}$$

$$\hat{\rho}_0 = -\hat{T}_0$$
as
$$r \to \infty$$

$$\hat{u}_{z0} \to 0$$

$$\hat{u}_{\theta 0} \to 0$$

$$\hat{T}_0 \to 0$$

$$\hat{Y}_0 \to 0$$

The solutions to the energy and species equations are:

$$\hat{T}_0 = \sum_{n=0}^{\infty} \frac{D_n}{\eta^{n+1}} P(\cos \theta)$$

$$\hat{Y}_0 = \sum_{n=0}^{\infty} \frac{S_n}{\eta^{n+1}} P(\cos \theta)$$

where  $P_n$  is Legendre polynomial of order n.

Again, matching the leading - order solutions in zone I with the leading - order solutions in zone II, we get:

$$\begin{split} &D_n = S_n = 0 \text{ for } n \ge 1 \\ &\hat{m}_0 = \tilde{m}_0 = m_0 \\ &D_0 = N_0 = -\frac{\text{ReSc } m_0}{\text{Le}} \big[ 1 - \big( T_s - L \big) \big] \\ &S_0 = V_0 = -\text{ReSc } m_0 \big( Y_s - 1 \big) \end{split}$$

The velocity profile can be obtained using the stream function approach:

$$\hat{u}_{\eta 0} = \frac{1}{\eta^2 \sin \theta} \frac{d\hat{\psi}}{d\theta}$$

$$\hat{u}_{\theta 0} = -\frac{1}{\eta \sin \theta} \frac{d\hat{\psi}}{d\eta}$$
Substituting the above into the momentum equation and taking the

curl yields:

$$D^2\hat{\psi} = \sin^2\theta \left(\frac{D_0}{\eta}\right)$$

where

$$D = \frac{\partial^2}{\partial \eta^2} + \frac{\sin \theta}{\eta^2} \frac{\partial}{\partial \theta} \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \right)$$

The general solution is found to be:

$$\hat{\psi} = \left(\frac{B_1}{\eta} + B_2 \eta + B_3 \eta^2 + B_4 \eta^4 + \frac{D_0}{8} \eta^3\right) \sin^2 \theta$$

$$\hat{\mathbf{u}}_{\eta 0} = 2 \left( \frac{\mathbf{B}_1}{\eta^3} + \frac{\mathbf{B}_2}{\eta} + \mathbf{B}_3 + \mathbf{B}_4 \eta^2 + \frac{\mathbf{D}_0}{4} \eta \right) \cos \theta$$

$$\hat{\mathbf{u}}_{\theta 0} = \left(\frac{B_1}{\eta^3} - \frac{B_2}{\eta} - 2B_3 - 4B_4\eta^2 - \frac{3D_0}{8}\eta\right) \sin\theta$$

where B<sub>1</sub>, B<sub>2</sub>, B<sub>3</sub> and B<sub>4</sub> are constants

From the principle of the minimum singularity (see Van Dyke), it can be concluded that the constant B<sub>4</sub> should equal to zero. The velocity profiles are as follows:

$$\hat{\mathbf{u}}_{z0} = 2 \left( \frac{B_1}{\eta^3} + \frac{B_2}{\eta} + \frac{D_0}{4} \eta \right) \cos \theta$$

$$\hat{\mathbf{u}}_{\theta 0} = \left( \frac{B_1}{\eta^3} - \frac{B_2}{\eta} - \frac{3D_0}{8} \eta \right) \sin \theta$$

As was found in zone (II), the velocity field in zone (III) increases without bound as  $\eta$  increases. Again, this implies that analysis of at least one other zone is required to complete the analysis of this problem.

# DISCUSSION AND CONCLUSIONS

The nondimensionalization of the equations produced the result that the variable & characterizes the influence of gravity. When & is small, the flowfield is momentum-dominated near the surface of a droplet, with buoyancy subdominant. Since velocities decrease as fluid particles travel away from the droplet surface, buoyancy effects eventually become as important as momentum and viscous effects.

For  $\varepsilon \to 0$ , the analysis of zone (I) indicates that the flowfield near the droplet surface will be spherically-symmetrical to leading order. Correction terms are of order  $\varepsilon$  in this zone for all variables. A result of this analysis is the expectation that droplet vaporization rates should increase linearly with  $\varepsilon$  so long as  $\varepsilon << 1$ . Eventually, however, as r increases, the solution in zone (I) breaks down, which leads to the analysis of zone (II).

zone (II), the leading-order solutions are spherically symmetrical, while the correction terms include nonzero velocities in the 0-direction that tend to turn the flow in the direction of the gravity Eventually, as the coordinate z becomes large in one (II), the asymptotic correction term becomes as large in magnitude as the leading-order term. This is interpreted as an indication that a stagnation point will likely exist near the outer boundary of zone (II). To reinforce this idea of the existence of a stagnation point, inspection solutions obtained for zone (III) indicates that the velocities in this zone will in general be "downward," that is, they will be in the direction of the gravity vector; these downward velocities should produce a stagnation point along a symmetry line of the flowfield. Based upon the scalings derived in this paper,

dimensionless radial location  $(r_s)$  of the stagnation point is expected to scale as  $r_s \approx \epsilon^{-1/3}$ . A qualitative sketch of the expected flowfield in the vicinity of a droplet is shown in Fig. (1). Only the streamlines on one side of the droplet are shown; the streamlines on the other side are symmetrical.

As noted previously, further analysis is required to provide a description of the flows outside of zone (III); this research is presently underway and will be reported at a later time. Based upon previous analyses<sup>8,9,13</sup>., it is expected that a plume-like structure will eventually be formed below the droplet; boundary-layer theory likely be used to analyze the plume structure. However, analysis of the transition from the flowfield near the droplet to the plume-like flow is expected to be challenging. A sketch of the expected flowfield far away from a droplet is shown in Fig. (2). Only the streamlines on one side of the droplet are shown; the streamlines on the other side are symmetrical.

### **ACKNOWLEDGEMENT**

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<sup>&</sup>lt;sup>2</sup> Fendell, F. E. Spankle, M. L., and

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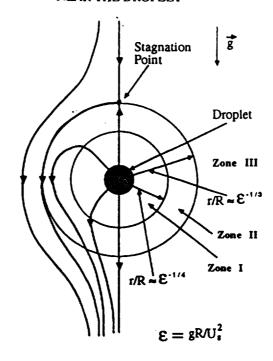


Figure 1. Qualitative sketch of the flowfield near a gasifying droplet.

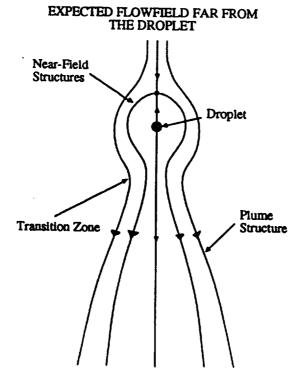


Figure 2. Qualitative sketch of the flowfield far from a gasifying droplet.

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